**Name:**

**Math 10a Quiz 9 Solutions**
November 13, 2013

1. (4 points) The following game is proposed to the management at a casino: A participant rolls a fair six-sided die. If the die lands on an even number, then the participant wins that amount in dollars. If the die lands on an odd number, then the participant loses twice that amount in dollars.

   Would this game be profitable for the casino? Back up your answer quantitatively.

   Let $X$ be the amount the player wins with one roll. Then

   $$E(X) = \frac{1}{6}(-2 \cdot 1) + \frac{1}{6}(2) + \frac{1}{6}(-2 \cdot 3) + \frac{1}{6}(4) + \frac{1}{6}(-2 \cdot 5) + \frac{1}{6}(6) = -2$$

   so the player loses two dollars on average per roll. Since the player is losing the money, the casino is gaining the money, so the game will be profitable after many people play it.

2. Let $X$ be a binomial random variable that counts the number of successes in 10 independent trials, where each trial has a $p$ chance of success.

   (a) (1 points) What is the pmf of $X$?

$$P(X = x) = \binom{10}{x} p^x (1-p)^{10-x}, \ x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

   (b) (1 points) Derive the pmf with respect to the parameter $p$.

$$\frac{d}{dp}\binom{10}{x} p^x (1-p)^{10-x} = \binom{10}{x}\left(xp^{x-1}(1-p)^{10-x} - (10-x)p^x(1-p)^{10-x-1}\right).$$

   (c) (2 points) What is $P(X \leq 3)$? (do not worry about simplifying your answer)

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$= \binom{10}{0} p^0 (1-p)^{10-0} + \binom{10}{1} p^1 (1-p)^{10-1} + \binom{10}{2} p^2 (1-p)^{10-2} + \binom{10}{3} p^3 (1-p)^{10-3}$$

   (d) (2 points) A biology graduate student has a collection of 60 lab mice, precisely 15 of which have a certain genetic mutation $M$. She randomly selects 10 different mice from the collection of 60 mice and (erroneously) writes down in her notes that the number of mice with the mutation $M$ in her sample can be modeled using the random variable $X$ with a parameter $p = 1/4$.

   Why does this model not work?

   If $X$ were to model this situation the trials would have to be selection of a mouse from the 60 mice. But these trials are not independent. For example, if the first mouse picked has a mutation then the probability that the next mouse has a mutation is slightly less than 1/4 (14/59, to be precise). The binomial model only applies in situations where the trials are independent. Sometimes, if the population size is vastly larger than the sample size (for example, polling 100 random people out the 300 million in this country), then selections without replacement are approximately independent, but the population to sample size ratio (6:1) is just not large enough for that to be the case here.